Exam. Code : 121201
Subject Code : 3954

B.A./B.Sc. 1st Semester (Batch 2023-26) (CBGS)

MATHEMATICS (Algebra)

Paper—I

Time Allowed—3 Hours] [Maximum Marks—100

Note :—Attempt **FIVE** questions in all, selecting at least **ONE** question from each section. The fifth question may be attempted from any section. All questions carry equal marks.

SECTION—A

1. (a) Examine the consistency of the following equations and if consistent, find the complete solution :

$$x - y + 2z = 4$$
$$3x + y + 4z = 6$$
$$x + y + z = 1$$

(b) Prove that every skew-symmetric matrix of odd order has rank less than its order.

2. (a) Prove that system of vector $x = (1, 5, 2)$; $y = (0, 0, 1)$; $z = (1, 1, 0)$ of $V_3(R)$ is L.I.

(b) Find the rank of AB, A + B when :

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}, B = \begin{bmatrix} -1 & -2 & -1 \\ 6 & 12 & 6 \\ 5 & 10 & 5 \end{bmatrix}$$

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SECTION—B

3. Verify Cayley Hamilton theorem for the matrix $A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 2 & 0 \\ 2 & -1 & 0 \end{bmatrix}$. Hence find A^{-1}.

4. Find the characteristic roots and their corresponding characteristic vectors for the matrix $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$.

SECTION—C

5. (a) Reduce the following quadratic forms to sum of the squares by linear transformation :

$$2x^2 + 9y^2 + 6z^2 + 8xy + 8yz + 6zx.$$

(b) Explain semi-definite and indefinite matrix.

6. (a) Reduce the diagonal matrix by congruent transformations :

$$A = \begin{bmatrix} 1 & 2 & -1 \\ -6 & 0 & 9 \\ -1 & 3 & -4 \end{bmatrix}$$

(b) Explain congruence of skew-symmetric matrices with example.

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SECTION–D

(a) Explain Ferrari's methocof solving a biquadratic equation.

(b) Use Descartes's metho(to solve :

$$x^4 - 10x^2 - 20x - 16 = 0.$$

(a) Use Cardan's method o solve :

$$x^3 - 3x^2 - 10x + 24 = 0.$$

(b) Reduce the equation $2x^3 - 3x^2 + 10x - 4 = 0$ to the form $Z^3 + 3Hz + G = 0$ where H and G are the constants.

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