

Exam. Code : 121201

Subject Code : 502113

B.A./B.Sc. 1st Semester (Batch 2025-28/29) (CBGS)

MATHEMATICS

(Algebra)

Time Allowed—3 Hours] [Maximum Marks—100

Note :— Attempt **FIVE** questions in all, selecting at least **ONE** question from each section. The fifth question may be attempted from any section. All questions carry equal marks.

SECTION—A

1. (a) Find the conditions on h and k so that the rank

of the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & h-2 & 2 \\ 0 & k-1 & h+2 \\ 0 & 0 & 3 \end{bmatrix}$ is less than 3.

Under each condition, find the rank of the matrix.

10

- (b) Find the non-singular matrices P and Q such that PAQ is in the normal form, where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 3 & 1 & 1 \end{bmatrix}. \quad 10$$

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2. (a) Investigate for what values of λ, μ the simultaneous equations

$$x + y + z = 6$$

$$x + 2y + 3z = 10$$

$$x + 2y + \lambda z = \mu$$

have:

- (i) No solution.
 - (ii) A unique solution.
 - (iii) An infinite number of solutions. 10
- (b) Solve the following system of equations

$$4x + 5y + 6z = 0,$$

$$5x + 6y + 7z = 0,$$

$$7x + 8y + 9z = 0. \quad 10$$

SECTION—B

3. (a) Show that every square matrix is uniquely expressible as the sum of a Hermitian matrix and a Skew-Hermitian matrix. 10
- (b) Find the eigenvalues and corresponding

eigenvectors for the matrix $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$. 10

4. (a) State Cayley-Hamilton Theorem and verify it for

the matrix $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$. 10

- (b) Using Cayley-Hamilton Theorem find A^6 if

$A = \begin{bmatrix} 2 & 1 \\ 5 & -2 \end{bmatrix}$. 10

SECTION—C

5. (a) Define a quadratic form. Reduce the quadratic form

$Q(x, y, z) = 3x^2 + 3y^2 + 3z^2 + 2xy - 2yz - 2zx$
to its canonical form by an orthogonal transformation and find its rank, index, and signature. 10

- (b) Find the nature (definite, semi-definite, or indefinite) of the quadratic form

$Q(x, y, z) = 2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx$. 10

6. (a) What are definite, semi-definite, and indefinite quadratic forms? Give example for each. 10

- (b) Define congruent matrices. Show that congruence is an equivalence relation on the set of all square matrices. 10

SECTION—D

7. (a) Using Ferrari's method, find the roots of the biquadratic equation $x^4 - 10x^2 + 9 = 0$. 10
- (b) Explain Cardan's method for solving a cubic equation. Solve the equation $x^3 - 6x^2 + 11x - 6 = 0$ using this method. 10
8. (a) Derive Vieta's formulas for a general cubic equation $ax^3 + bx^2 + cx + d = 0$. Using these, find the relation between the roots and coefficients of the equation $2x^3 - 3x^2 - 8x + 12 = 0$. 10
- (b) State Descartes' Rule of Signs. Apply it to determine the possible number of positive and negative real roots of the polynomial $f(x) = 2x^5 - 5x^4 + 4x^3 - x^2 + 6x - 3$. 10